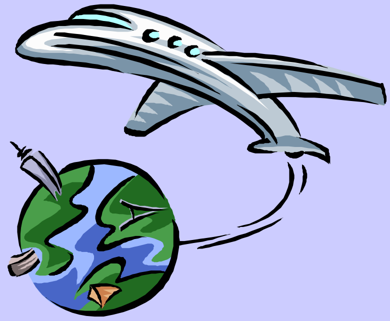

A Computational Study of Benders Decomposition for the Integrated Aircraft Routing and Crew Scheduling Problem

A. Mercier, J.-F. Cordeau and F. Soumis
GERAD and École Polytechnique de Montréal

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Tactical Planning

Flight Scheduling

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Fleet Assignment

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Aircraft Routing

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Crew Scheduling

↓

Individual Monthly Schedules

Tactical Planning

Flight Scheduling

↓

Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling

} short connections

↓

Individual Monthly Schedules

Integrated Model

Benefits and Drawback

Benefits :

- Increased flexibility
- Lower costs

Drawback :

- Large size of model

Instance	Legs	Constraints	Short Connections	SC in % of Constraints
D95	196	519	89	17.15%
D9S	523	1622	502	30.95%
MD80LA	707	2647	1183	44.69%

For the largest instance:

The integrated model contains almost 4 times the number of constraints of each of the individual problems.

Integrated Routing and Scheduling

Literature Review

Partial Integration

Klabjan et al. (2002)

Crew scheduling with plane count constraints and time windows

Total Integration

Cordeau et al. (2001)

Complete formulation solved by Benders decomposition

Cohn and Barnhart (2003)

Extended crew scheduling model with aircraft solutions as variables (unique and maximal maintenance feasible SC set)

Integrated Model

Robustness

- Upper bound on number of short connections used

- Penalties on short connections

- Penalties on restricted connections

  more linking constraints

Integrated Model

Assumptions

- Daily problem

- Aircraft routing constraints

    Number of aircraft

    Maintenance (routine checks)

- Approximate crew costs

    Waiting and deadhead costs

- Crew scheduling constraints

    Duty: Max on duty time, flight time, nb. of landings

    Min on avg. duty time

    Pairing: Max on nb. of duties, nb. of days

Solution Methodology

3-phase Approach

PHASE I : Solve LP relaxation of integrated model with Benders Decomposition.

Reverse natural solution sequence:

Benders MP : Crew Scheduling

Benders SP : Aircraft Routing

Both solved by Column Generation

PHASE II : Introduce integrality constraints on MP variables and solve the resulting MIP by generating additional cuts.

PHASE III : Introduce integrality constraints on SP variables and solve SP with MP variables held fixed.

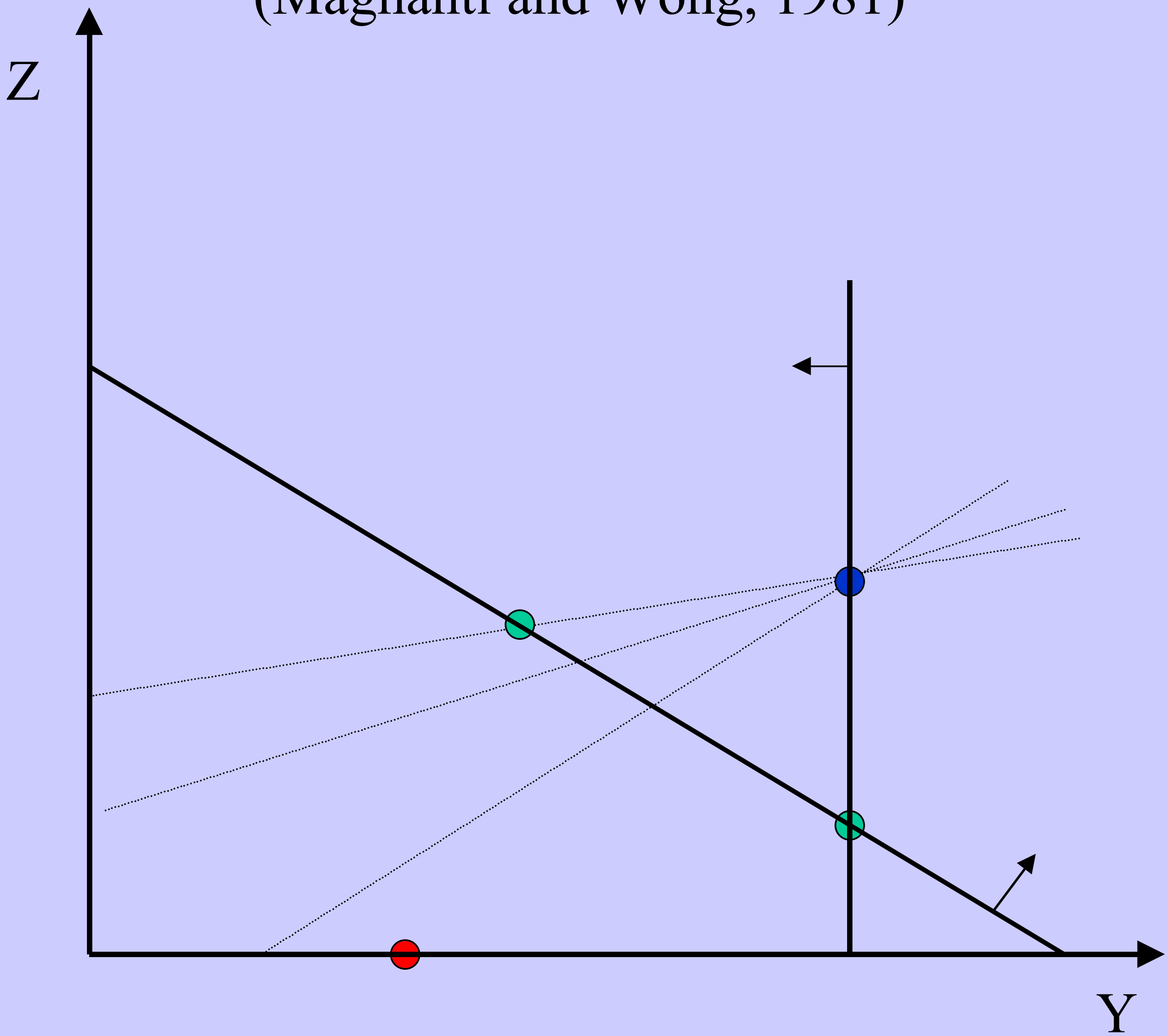
If infeasible: go back to phase II

An additional constraint is added to forbid the same set of SC to be chosen.

Solution Methodology

Strong Cuts

Pareto-Optimal Cuts
(Magnanti and Wong, 1981)



Solve an auxiliary SP at every iteration in Phase I and II

Solution Methodology

Heuristic Branch-and-Bound

Branching is done mainly on CG MP variables :

- At each node of the tree, some variables having a value close to 1 are sequentially fixed to 1 and the resulting LPs are evaluated.

- The strategy is modified towards the end of the tree where complete enumeration is performed.

Variance in solution quality and CPU time decreases

Computational Experiments

Description of Data Sets

Daily fleet assignment solutions from two major airlines

Instance	Number of Legs	Number of Aircraft	Number of Constraints
B767R	151	38	579
B757A	184	60	564
A320B	258	80	877
D9SA	525	116	1893
D9SB	508	116	2217
B767S	510	130	2053
MD80L	707	143	3537

Computational Experiments

Summary of Results

Phase I Results

Robust Integrated Model

767SA 510 legs	Optimality Tolerance	MP Aircraft	MP Crew	% Change	MP Crew P.-O. Cuts	% Change
Cuts	1 %	28	25	-10.71%	2	-92.86%
CPU (min)		50.26	33.11	-34.12%	7.37	-85.34%
Cuts	0.1%	286	165	-42.31%	3	-98.95%
CPU (min)		498.31	165.98	-66.69%	11.14	-97.76%

Optimality Tolerance: Benders stopping criterion

% Change: MP Crew vs MP Aircraft

MP Crew P.-O. Cuts vs MP Aircraft

Computational Experiments

Summary of Results

3-Phase Results - Integrated Model

	MP Aircraft	MP Crew P.-O. Cuts	% Change
B757A (184 legs)			
Gap	1.34%	1.29%	
Total CPU (min)	15.41	6.28	-59.25%
B767R (151 legs)			
Gap	1.43%	1.45%	
Total CPU (min)	7.02	1.86	-73.50%
A320B (258 legs)			
Gap	1.73%	1.81%	
Total CPU (min)	99.12	50.18	-49.37%
D9SA (525 legs)			
Gap	0.05%	0.00%	
Total CPU (min)	78.22	20.63	-73.63%
D9SB (508 legs)			
Gap	0.87%	0.87%	
Total CPU (min)	530.80	54.45	-89.74%
B767S (510 legs)			
Gap	2.85%	1.78%	
Total CPU (min)	373.6	42.05	-88.74%
MD80L (707 legs)			
Gap	1.20%	0.68%	
Total CPU (min)	1171.97	81.55	-93.04%

Computational Experiments

Summary of Results

3-Phase Results - Robust Integrated Model

	MP Aircraft	MP Crew P.-O. Cuts	% Change
B757A (184 legs)			
Gap	1.73%	1.49%	
Total CPU (min)	44.03	4.76	-89.19%
B767R (151 legs)			
Gap	1.21%	1.23%	
Total CPU (min)	21.07	5.45	-74.13%
A320B (258 legs)			
Gap	1.72%	1.89%	
Total CPU (min)	557.19	7.95	-98.57%
D9SA (525 legs)			
Gap	0.18%	0.18%	
Total CPU (min)	222.15	44.99	-79.75%
D9SB (508 legs)			
Gap		1.17%	
Total CPU (min)	+ 30 hours	110.17	
B767S (510 legs)			
Gap		1.00%	
Total CPU (min)	+ 30 hours	52.07	
MD80L (707 legs)			
Gap		1.53%	
Total CPU (min)	+ 30 hours	173.91	