

A Robust Fleet Assignment Model with Hub Isolation and Short Cycles

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Robust Planning

- Traditional airline planning models assume that every flight takes off and lands as planned.
- Objective functions typically reduce slack in the schedule and ignore operations.
- By considering disruptions in the schedule, we can construct plans that perform well in operations.

Robust Planning Literature

- Ehrgott, M. and Ryan, D. M. (2003). Constructing Robust Crew Schedules with Bicriteria Optimization, To appear in *Journal for Multi-Criteria Decision Analysis*,
- Schaefer, A. J., Johnson, E. L., Kleywegt, A. J. and Nemhauser, G. L. (2001). Airline Crew Scheduling Under Uncertainty, *Technical Report TLI/LEC-01-01*, Georgia Institute of Technology, Atlanta, GA.
- Schaefer, A. J. (2000). *Airline Crew Scheduling Under Uncertainty*. Ph.D. Dissertation, Georgia Institute of Technology, Atlanta, GA, The Logistics Institute.
- Yen, J. W. (2000). *A Stochastic Programming Formulation of the Stochastic Crew Scheduling Problem*. Ph.D. Dissertation, University of Michigan, Ann Arbor, MI.
- Ageeva, Y. (2000). *Approaches to Incorporating Robustness into Airline Scheduling*. M.Eng. Thesis, Massachusetts Institute of Technology, Cambridge, MA, Department of Electrical Engineering and Computer Science.

Robust Fleet Assignment

- Traditional fleet assignment models:
 - Minimize passenger spill and operating cost.
 - Assume that every flight takes off and lands as planned.
- Robust fleet assignment model:
 - Uses a string fleet assignment model from Barnhart et al.
 - Barnhart, C., Boland, N., Clarke, L., Johnson, E. L. and Nemhauser, G. L. (1998). Flight string models for aircraft fleet assignment and routing, *Transportation Science* 32: 208–220.
 - Reduces hub connectivity and embeds cancellation cycles.
 - Provides better results in operations.

Hub-and-Spoke Networks

- A *hub-and-spoke* network is the structure of an airline flight schedule in which a large percentage of the legs go into or out of a small subset of stations called *hubs*. *Spokes* have limited activity.
- Hub-and-spoke networks allow passengers to fly from an origin to a destination with few intermediate stops.
- Hub-and-spoke networks are highly sensitive to disruptions at hubs.
- *Hub connectivity* is the number of legs in a rotation that can be found in a subroute that begins at a hub, ends at a different hub, and only stops at spokes in between.
- Reducing hub connectivity makes operations at a hub less sensitive to disruptions at other hubs.

Hub Connectivity

Assignment 1			
Flight	Departure Station	Arrival Station	Plane
21	Boston	Pittsburgh	1
22	Pittsburgh	Baltimore	1
23	Baltimore	Pittsburgh	1
24	Pittsburgh	Boston	1
25	Boston	Pittsburgh	2
26	Pittsburgh	Baltimore	2
27	Baltimore	Pittsburgh	2
28	Pittsburgh	Boston	2

- Flight schedule has hubs in Baltimore and Boston.
- Hub connectivity of assignment 1 is 8 legs.

Hub Connectivity

Assignment 2			
Flight	Departure Station	Arrival Station	Plane
21	Boston	Pittsburgh	1
24	Pittsburgh	Boston	1
25	Boston	Pittsburgh	1
28	Pittsburgh	Boston	1
22	Pittsburgh	Baltimore	2
23	Baltimore	Pittsburgh	2
26	Pittsburgh	Baltimore	2
27	Baltimore	Pittsburgh	2

- Flight schedule has hubs in Baltimore and Boston.
- Hub connectivity of assignment 2 is 0 legs.

Cancellation Cycles

- A *cancellation cycle* is a sequence of flight legs that begins and ends at the same station within a rotation.
- Cancelling cycles maintain aircraft flow balance.
- Cancelling short cycles can provide slack during recovery.
 - Suppose an aircraft requires several hours of unscheduled maintenance. Cancelling a short cycle could provide the necessary slack in the rotation to repair the aircraft.
 - Suppose an airline needs to cancel several legs departing from or arriving at an airport experiencing inclement weather. The airline could cancel several short cycles that include this airport.

String Fleet Assignment Model

A *string* is a sequence of legs flown by the same aircraft.

$$z_{FAM} = \min \sum_{s \in S} \sum_{j \in J} c_{js} X_{js} \quad (1)$$

$$\sum_{s \in S(f)} \sum_{j \in J} X_{js} = 1 \quad \forall f \in F \quad (2)$$

$$\sum_{\{ogt_{ot}\} \in S} X_{jogt_{ot}} + Y_{jgt^{-}t} - \sum_{\{gdt_{ot}\} \in S} X_{jgdt_{ot}} - Y_{jgtt^{+}} = 0 \quad \forall \{jgt\} \in N \quad (3)$$

$$\sum_{s \in O(j)} X_{js} + \sum_{g \in G} Y_{jgt_{nt_1}} \leq P(j) \quad \forall j \in J \quad (4)$$

$$Y_{jgtt^{+}} \geq 0 \quad \forall \{jgt\} \in N \quad (5)$$

$$X_{js} \in \{0, 1\} \quad \forall s \in S, j \in J \quad (6)$$

Let \mathcal{X} be the set of X such that there exists Y and (X, Y) satisfy equations (2) through (6).

Hub Based Fleet Assignment

- Each string of a hub based fleet assignment model starts at a hub, flies to a non-negative number of spokes, and ends at a hub.

- The sequence of stations visited by a string is

$$(\mathcal{H}_0, \mathcal{S}_1, \dots, \mathcal{S}_{n-1}, \mathcal{H}_n).$$

- The strings that start and end at the same hub, $\mathcal{H}_0 = \mathcal{H}_n$, are cancellation cycles.
- An *acyclic string* is such that $\mathcal{H}_0 \neq \mathcal{H}_n$.
- For each $f \in \mathcal{HH}$, leg f is included in exactly one string consisting of only leg f .

Hub Based Fleet Assignment

● Let (V^*, U^*) be the set of acyclic and cyclic strings of a hub based fleet assignment solution. Then,

1. $|V^*| + |U^*| = |\mathcal{HH}| + |\mathcal{HS}|.$

2. $|V^*| \leq \frac{|\mathcal{HH}| + |F(V^*)|}{2}.$

3. $|\mathcal{HS}| \geq |U^*| \geq |\mathcal{HS}| - \frac{|F(V^*)| - |\mathcal{HH}|}{2}.$

● Let $U_2^* \subseteq U^*$ be the set of cancellation cycles with only two legs, called *out-and-backs*. Then,

$$|\mathcal{HS}| \geq |U_2^*| \geq |\mathcal{HS}| - |\mathcal{SS}| - \frac{|F(V^*)| - |\mathcal{HH}|}{2}.$$

Hub Based Fleet Assignments

- For each string $s \in S$ and for each fleet $j \in J$, let

$$h_{js} = \begin{cases} |F(s)| & \text{if } s \text{ is acyclic,} \\ 0 & \text{otherwise.} \end{cases}$$

- A lower bound on the hub connectivity is the objective value of the *hub isolation problem*, (HI),

$$z_{HI} = \min \left\{ \sum_{s \in S} \sum_{j \in J} h_{js} X_{js} \mid X \in \mathcal{X} \right\}.$$

- The *hub connectivity problem* is, (HC),

$$z_{HC} = \max \left\{ \sum_{s \in S} \sum_{j \in J} h_{js} X_{js} \mid X \in \mathcal{X} \right\}.$$

Hub Based Fleet Assignments

• We minimize cost and constrain hub connectivity, (MCHCC),

$$\begin{aligned} \min \quad & \sum_{s \in S} \sum_{j \in J} c_{js} X_{js} \\ & \sum_{s \in S} \sum_{j \in J} h_{js} X_{js} \leq \varsigma, \\ & X \in \mathcal{X}. \end{aligned}$$

• We isolate hubs subject to a cost constraint, (HICC),

$$\begin{aligned} \min \quad & \sum_{s \in S} \sum_{j \in J} h_{js} X_{js} \\ & \sum_{s \in S} \sum_{j \in J} c_{js} X_{js} \leq \varrho, \\ & X \in \mathcal{X}. \end{aligned}$$

Flight Schedule Examples

Characteristics of flight schedules A, B, and C.

Total	Schedule A	Schedule B	Schedule C
Daily legs	2558	910	573
Hubs	8	7	5
Hub-to-spoke legs	1055	398	251
Hub-to-hub legs	418	107	47
Spoke-to-hub legs	1055	398	251
Spoke-to-spoke legs	30	7	24

Fleet Examples

Characteristics of the fleets assigned to flight schedule A.

Fleet	Number of Aircraft	Aircraft Capacity (in Passengers)	Operating Cost per Block Hour (in Dollars)
A1	75	189	2544
A2	80	130	2043
A3	70	126	1850
A4	70	149	1850
A5	50	150	2140
A6	50	139	1799
A7	70	172	2046
A8	75	139	1789
A9	85	172	1844

Fleet Examples

Characteristics of the fleets assigned to flight schedules B and C.

Fleet	Number of Aircraft	Aircraft Capacity (in Passengers)	Operating Cost per Block Hour (in Dollars)
B1	96	135	2000
B2	70	170	2544
B3	32	150	2532
C1	51	405	5024
C2	51	175	1822
C3	47	301	3315

Hub Connectivity Versus Cost

Results of MCHCC for flight schedule B.

ζ	z_{FAM}	$F(V^*)$	$ U^* $	$ U_2^* $
289	11131388	289	307	302
292	11130531	291	306	301
295	11129870	295	304	299
300	11129718	299	302	297
305	11129041	303	300	295
325	11128450	315	294	289
350	11127930	349	277	272
400	11127907	397	254	250

Hub Connectivity Versus Cost

Results of MCHCC for flight schedule C.

ζ	z_{FAM}	$F(V^*)$	$ U^* $	$ U_2^* $
197	7050792	197	179	161
200	7042123	200	178	161
205	7036917	205	176	160
210	7034535	210	174	159
220	7031212	220	169	154
230	7031212	230	165	152

Assignments for Schedule A

Characteristics of assignments of schedule A to fleets A1 through A9.

Assign- ment	Problem Type	ς	ϱ	z_{FAM}	$ F(V^*) $	$ U^* $	$ U_2^* $
AFAM	FAM	-	-	23415805	1200	664	644
AHI	HI	-	-	25806826	586	971	951
AMCHCC	MCHCC	586	-	23449931	586	971	952
AHICC	HICC	-	23500000	23499820	586	971	953
AHC	HC	-	-	25565659	1672	428	412

Assignments for Schedule B

Characteristics of assignments of schedule B to fleets B1 through B3.

Assign- ment	Problem Type	ς	ϱ	z_{FAM}	$ F(V^*) $	$ U^* $	$ U_2^* $
BFAM	FAM	-	-	11127926	395	255	251
BHI	HI	-	-	11854121	289	307	303
BMCHCC	MCHCC	289	-	11131388	289	307	302
BHICC	HICC	-	11128500	11128450	315	289	294
BHC	HC	-	-	11776300	485	212	211

Assignments for Schedule C

Characteristics of assignments of schedule C to fleets C1 through C3.

Assign- ment	Problem Type	ς	ϱ	z_{FAM}	$ F(V^*) $	$ U^* $	$ U_2^* $
CFAM	FAM	-	-	7031212	238	161	148
CHI	HI	-	-	10389713	197	179	161
CMCHCC	MCHCC	197	-	7050792	197	179	161
CHICC	HICC	-	7035000	7034924	209	175	161
CHC	HC	-	-	10118898	299	132	122

Simulation Results for Schedule A

Assign- ment	OT +15	OT +60	Late- ness	Can %	Que %	Swaps
AFAM	69.93	88.89	16.103	4.94	1.27	821568
AHI	71.04	89.83	15.691	4.35	1.26	851545
AMCHCC	70.54	89.43	15.813	4.65	1.21	799732
AHICC	70.41	89.26	16.232	4.76	1.22	809498
AHC	70.12	88.80	15.787	5.22	1.33	830561

Optimal Fleet Assignment Analysis

- An *inter-fleet swap* occurs when a controller changes the fleet type of a leg in operations.
- Operations controllers rarely swap across fleets because of disruptions, so a controller can be constrained by the initial fleet assignment.
- By assuming that every leg is flown by the same fleet, we can estimate the operations of an optimal fleet assignment.

Simulation Results for Schedule B

Assign- ment	OT +15	OT +60	Late- ness	Can %	Que %	Swaps
BFAM	69.92	88.95	17.940	4.16	1.17	208101
BHI	69.83	89.12	18.002	3.90	1.18	210567
BMCHCC	69.97	89.08	18.010	3.97	1.17	181061
BHICC	69.96	89.07	17.521	4.00	1.16	202461
BHC	69.57	88.58	18.22	4.40	1.18	198576
BOPT	71.21	88.68	17.356	3.83	1.23	186174

Simulation Results for Schedule C

Assign- ment	OT +15	OT +60	Late- ness	Can %	Que %	Swaps
CFAM	72.33	90.76	14.126	3.51	1.13	133809
CHI	72.61	91.10	14.425	2.87	1.21	125717
CMCHCC	72.56	90.94	13.735	3.54	1.10	80941
CHICC	72.49	90.88	13.986	3.47	1.12	103113
CHC	72.41	90.97	14.453	3.01	1.21	156639
COPT	76.23	92.19	12.337	2.63	1.23	97596

Future Research

- The airline environment is continually changing.
- Robust planning initiatives include:
 - Increased block time.
 - Hub isolation.
 - Overlapping aircraft routes and crew pairings.
 - Stochastic aircraft rotation, crew pairings, and passenger itineraries.
- Recovery initiatives include:
 - Integrated recovery.
 - Robust recovery.